Tesla Turbine for Pico Hydro Applications

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Rural electrification is needed to improve the livelihoods of individuals that are located where centralized power grids do not reach. Pico hydro, hydro systems of 5 kW capacity or less, can address this need at relatively low cost and with virtually no negative environmental or social impacts. An alternative and less complex design for a pico hydro generator rotor, proposed by Nikola Tesla, uses the viscosity of fluids to efficiently transfer the fluid energy to the rotor, resulting in usable work. Numerous analytical and experimental investigations have been performed to characterize the flow within the Tesla turbine design. It has been found that efficiencies reported by Tesla have not been reproduced by others. Inefficiencies are often attributed to the losses due to flows through the nozzle and outlet. Tesla turbines will not be considered competitive in the general market until efficiencies surpass that of conventional designs and power densities can be improved. That said, the relatively simple design of the Tesla turbine lends itself well to small scale power projects since it can be easily manufactured and maintained locally. Debris in water sources is a significant concern in small scale hydro systems. The disk rotors are not sensitive to abrasion and can continue to perform under these types of flows. A preliminary design of a Tesla turbine for pico hydro applications has been undertaken employing Rice’s analytical method. The investigation yielded a design capable of generating 300 W under conditions of 20 m head and 2.5 L/s flow. The efficiency of the preliminary turbine design was near 80% but it is believed that the bulk of efficiency losses will be found in inlet and exhaust flows, which are not considered in this analysis. Challenges remain for the application of Tesla turbines to pico hydro generation.

Nomenclature

- \( b \) = distance between adjacent disks
- \( f \) = Darcy friction factor
- \( g \) = gravity
- \( H \) = pressure head
- \( P \) = power
- \( p \) = pressure
- \( \Delta p_n \) = pressure change across nozzle flow
- \( \Delta p_r \) = pressure change across rotor disc from \( r_o \) towards disk centre
- \( \Delta p_t \) = \( \Delta p_n + \Delta p_r \), total pressure change across rotor and nozzle
- \( Q \) = volumetric flow rate of single disk spacing
- \( \bar{Q} \) = total volumetric flow rate
- \( r_o \) = outer radius
- \( r_i \) = inner radius
- \( T \) = torque
- \( v_o \) = outer tangential velocity
- \( v_i \) = inner tangential velocity
- \( W \) = work per unit mass
- \( x \) = \( r/r_o \), dimensionless radial coordinate
- \( y \) = \( v/v_o \), dimensionless tangential velocity component
- \( \varepsilon \) = equivalent roughness
- \( \eta \) = efficiency
- \( \mu \) = dynamic viscosity
- \( \rho \) = density
- \( \Omega \) = angular velocity
I. Pico Hydro

Rural electrification enhances welfare through increased productivity, health, media access, and education. Studies by International Evaluation Group of the World Bank (2008) have demonstrated this empirically and have also found that the benefits from rural electrification outweigh the cost of investment.

Pico hydro systems harness the energy in flowing water at capacities smaller than 5 kW. They are recognized as a viable option to electrify remote areas with regards to economical, environmental, and social perspectives. Pico hydro yields the lowest generating cost amongst off-grid energy options. Unlike large scale hydropower, there is low environmental impact with pico hydro systems, mainly due to the exclusion of large water containment. Associated large civil works and the displacement of inhabitants are not required for the commissioning of pico hydro systems. In addition, off-gassing of green house gases as a product from decomposing organic material within large reservoirs (Gunkel, 2009) is avoided.

II. General Hydro Principles

Hydro power is driven by extracting the potential energy from water over a height difference. The energy in the water is converted to mechanical energy and can be used directly or converted again into electrical energy by means of a generator. The term head, \( H \), is the measure of pressure in the water. It refers to the actual height difference the water travels. Power, \( P \), is the energy converted over time or the rate of work being done.

The power that can be extracted from the water can be calculated with the following formula:

\[
P = \eta Q t \rho g \]

where \( \eta \) is efficiency, \( Q \) is total volumetric flow, \( \rho \) is density, and \( g \) is gravity.

Different types of turbines can be selected to best suit given heads and flows. Two main families of turbines include impulse and reaction type turbines.

**Impulse turbines** - Pressurized water is converted to high-speed water jets that transfer the kinetic energy by impacting the turbine blades or buckets causing rotation. Since the water jet acts in the open, the casing essentially acts as a splash guard. Examples of impulse turbines include Pelton wheel, Turgo wheel and Michell-Banki (crossflow) turbines.

**Reaction turbines** - Water flow is directed to create a pressure difference across the blades to create lift to rotate the turbine. Examples of reaction turbines include Kaplan and Francis turbines.

III. Tesla Turbine

For the application of pico hydro, several designs can be utilized. Common designs incorporate mechanisms such as pistons, paddles, vanes, and blades. An unconventional design was patented by Nikola Tesla in 1913 with the objective to develop a turbine with lower complexity, cost, and maintenance requirements than conventional mechanisms.

Within the literature, the Tesla turbine has been commonly referred as a boundary layer, multiple-disk, friction, or shear force turbine (Rice, 2003). The design consists of several, closely-spaced rigid disks set in parallel on a shaft (Figure 1). The co-rotating disks are centred and locked to the shaft. Located near the centre of the disks are orifices that allow for fluid exhaust in the axial direction. The disk-shaft assembly is set on bearings and enclosed within a cylindrical casing. An inlet into the casing is directed approximately tangentially (Tesla, 1913).

A. Theory of Operation

The working fluid enters the chamber through the inlet in the tangential direction and flows along the surface of the disk through the disk spacings. The flow path spirals towards the centre orifices, then exits axially through the outlet. Due to viscosity, the fluid adheres to the disks with the no-slip condition occurring directly adjacent to the disk surface and a velocity gradient forming throughout the working medium away from the surface. Through this phenomenon, some of the fluid energy is converted to mechanical work, causing the disks and shaft to rotate.
B. Current State of the Art

Since the 1950’s, there has been rejuvenated interest in the Tesla turbine. Numerous analytical methodologies have been proposed to characterize the flow within the rotor design. As flow through the multiple disk rotor can be laminar or turbulent, different mathematical methods have been devised for each of the flow regimes. Laminar flow calculations have been adequately established with different methods yielding fairly good agreement (Rice, 2003). For turbulent flows, several models exist, however, further experimentation and analyses are required to confidently determine the most applicable method (Rice, 2003).

On the experimental side, Tesla turbines have been constructed and tested by several investigators including Rice (1965), Hoya & Guha (2009), Armstrong, Beans, and Davydov & Sherstyuk (as cited by Guha & Smiley, 2009). Results have shown efficiencies in the range of 14.6 – 35.5 %, well below the 90% efficiency levels of present day large hydro turbines. Efficiency of 70% is typical for small hydro-turbines, with better designs achieving 80% or higher (Fraenkel, et al., 1991).

Lawn and Rice (1974) applied findings from experiments and analytical methods including earlier analyses of laminar flow between co-rotating disks (Boyd and Rice, 1968) to provide data for the design and performance of a Tesla turbine based on a volumetric flow rate parameter, Reynolds number and ratio of inner radius to outer radius. Several idealizations are associated with the performance calculation, including assumptions of laminar, incompressible Newtonian fluid flow and accounting only for losses due to the flow between disks. Losses resulting from the nozzle, nozzle-rotor interferences, rotor-housing disk friction, bearings, and exhaust passage pressure change are not accounted for.

In his review, Rice (2003) identified the nozzle and nozzle-rotor interaction as the main contributors to efficiency loss. Through experimentation and CFD analysis, Guha and Smiley (2009) have proposed an improved nozzle design that exhibits less than 1% drop in total pressure, reduced from the 13-34% in their original design. In addition, improvements to the inlet flow create more uniform distribution of the fluid to the disk spacings. The implication of these findings may be an improvement to the overall efficiencies of the Tesla turbine design.

C. Past Applications

Shortly after Tesla had presented his turbine design, Allis-Chalmers Manufacturing Company built three models with diameters of five feet and 500 kW capacities. They operated at 3600 rpm and reached a maximum capacity of 38% mechanical efficiency. Significant warping of the disks was witnessed. The project was stopped at this time with the main objections being the high rotational speeds and the lack of robustness of the disks (North, 1969).

In 1977, a 10-month pilot generation project had been conducted using the Tesla turbine design for the use in a hot salt-water mixture resource in Imperial Valley, CA, USA. The sub-surface turbine operated 24 hours per day. At the end of the investigation the turbine was dismantled and inspected for damage or wear due to particulate matter in the effluent. It was found that erosion was minimal and no repair or replacement of components was required (Jacobson, 1991).

The Tesla turbine concept has recently been adapted to wind turbines. The design includes an airfoil shaped spacer near the perimeter of the disk to induce lift and add further torque to the rotating shaft (Fuller, 2010).

IV. Tesla Turbines and Pico Hydro

Several aspects of the Tesla turbine cater well to pico hydro applications. The simple components and design of the Tesla rotor assembly are ideal for local capacity development throughout the entire life-cycle. With greater involvement of, and knowledge-transfer to, the end user, ownership is strengthened resulting in greater success (Williams, et al., 2000). This is achieved through sourcing of local parts, local manufacture, ease-of-maintenance, and reliability. In addition, through the placement of the turbine manufacture within a local setting, capital and maintenance costs are kept low for the user. This becomes an essential constraint when the users are low-income individuals.

Debris and abrasive two-phase flow is a constant threat to hydro generator performance. It is believed that the Tesla-type rotor disks are resistant to erosion (Rice, 2003) and therefore would make a good fit in a rural electrification setting.

To date, low efficiencies have been achieved with the Tesla design, greatly inhibiting the effectiveness of Tesla turbines for pico hydro as electrical demand tends to rise over time. In addition, power densities of current designs are low, and thus impact the transport and maintenance of these units. This must be considered as pico hydro systems are typically located away from households and in relatively inaccessible settings.

Tesla turbines show great promise with regard to pico hydro applications, however, further research must be conducted to improve efficiencies and achieve acceptable per kW costs.
V. Design Purpose

The remainder of this paper documents a preliminary design of a Tesla type turbine to be applied in a rural electrification context for low income communities. A power rating of 300 W has been selected to balance cost and material with consumption at the household level in a rural setting. This is suitable for the use of lighting and small media appliances. Typically, flow rate and pressure constrain the performance of a given turbine design; however this preliminary design will not be suited for specific site conditions. Instead it will be investigated if a preliminary design can achieve the target power levels using a realistic flow rate and pressure.

VI. Method

Rice (1965) had written on an experimental and analytical investigation of Tesla turbines. The analysis involved extensive amounts of data resulting in complete data sets not published. A program using Matlab was created to apply his work to this design. The program was validated by comparing with published results.

The analysis models the nature of flow between 2 co-rotating disks, and is then extended for multiple disk spacings. Several idealizations are considered:

- Frictionless flow occurs through the nozzles to the interspaced disks
- Uniform fluid is supplied at the disk outer radius
- Axially symmetric, 2-dimensional flow occurs in the plane of the disk
- Entire volume between disks is filled with water
- Smooth, parallel disks rotate with constant angular velocity
- Rotor housing does not restrict motion and is free of leaks
- Exhaust flow is uniform

Figure 2 illustrates the co-ordinate system used throughout the analysis.

Rice (1965) derived equations of motion of the fluid flow between disks based on a fluid element bounded by the solid disk spacing, with outer radius, \( r_o \), and thickness, \( b \). Forces considered are due to pressure and shear stress forces with body forces neglected to simplify the analysis. The non-dimensionalized equations of motion are as follows:

\[
\frac{dy}{dx} + \frac{y}{x} = \frac{fr_o^2 \Omega}{4bv_o} \left( \frac{v_o}{\Omega r_o} \right) y - x \left[ 1 + \left( \frac{2\pi br_o^2 \Omega}{Q} \right) x \left( \frac{v_o}{\Omega r_o} \right) (y - x) \right]^{\frac{1}{2}} = 0 \quad (2)
\]

\[
\frac{1}{\rho \Omega^2 r_o^2} \frac{dp_r}{dx} - \frac{Q}{2\pi br_o^2 \Omega} \left( \frac{1}{x^3} \right) \left[ \left( \frac{v_o}{\Omega r_o} \right)^2 \frac{y^2}{x} \right] - \frac{fr_o}{4b} \left( \frac{Q}{2\pi br_o^2 \Omega} \right) \left( \frac{1}{x^3} \right) \left[ 1 + \left( \frac{2\pi br_o^2 \Omega}{Q} \right) x^2 \left( \frac{v_o}{\Omega r_o} \right) (y - x) \right]^{\frac{1}{2}} = 0 \quad (3)
\]

where \( y \) is the tangential velocity ratio, \( x \) is the dimensionless radial co-ordinate, \( \Omega \) is the angular velocity, \( v_o \) is the outer tangential velocity, \( Q \) is the volumetric flow rate of single disk spacing, and \( p_r \) is the pressure at coordinate \( r \). Equation (2) describes the kinematic nature of the flow through the turbine and Eq. (3) describes the radial pressure change.

Included within the above equations are dimensionless turbine parameters:

- Friction factor, \( f \)
Within Rice’s analysis different combinations of values for the above turbine parameters were used for the analysis. Solutions for each combination were found. The Matlab program was used to solve the above ordinary differential equations; values of dimensionless velocity, $y$, and pressure change in the radial direction, $\Delta p_r$, were determined for given values of non-dimensional co-ordinate, $x$.

Total pressure change, $\Delta p_t$, through the turbine was determined by augmenting $\Delta p_r$ with a value of pressure change due to nozzles, $\Delta p_n$, derived from a general expression in dimensionless form:

$$
\frac{\Delta p_n}{\rho \Omega^2 r_o^2} = -\frac{1}{2} \left[ \frac{v_o}{\Omega r_o} \right]^2 + \left( \frac{Q}{2\pi br_o^3 \Omega} \right)^2
$$

(4)

$$
\frac{\Delta p_t}{\rho \Omega^2 r_o^2} = \frac{\Delta p_r}{\rho \Omega^2 r_o^2} + \frac{\Delta p_n}{\rho \Omega^2 r_o^2}
$$

(5)

Following this, values for dimensionless work, $W$ were determined using the following equation:

$$
\frac{W}{\Omega^2 r_o^2} = \frac{v_o}{\Omega r_o} \left(1 - xy\right)
$$

(6)

Efficiency, $\eta$, was then calculated as the ratio of dimensionless work to dimensionless total pressure change:

$$
\eta = \frac{W}{\Omega^2 r_o^2}
$$

(7)

For a given set of dimensionless turbine parameters, the turbine performance ($y$, $\eta$, and $\Delta p_t/\rho \Omega^2 r_o^2$) was plotted with respect to $x$, as seen in Figure 3. Corresponding values for $x$, $y$, and $\Delta p_t/\rho \Omega^2 r_o^2$, to the peak efficiency detail the optimized radial coordinate for the exhaust, and associated exit velocity and total pressure required, respectively. Several combinations of turbine parameters were solved.

By analyzing the turbine performances based on the different combinations of turbine parameters, trends were observed and used to determine appropriate parameters for the preliminary design. Design variables were then selected to calculate torque and power for a single disc spacing and used to evaluate the required total number of discs to achieve the target power.

VII. Design

As a starting point for the design, $r_o = 0.25$ m and $\Omega = 500$ rpm were selected. From these values, $b = 0.005$ m and $v_o = 13.09$ m/s were calculated based on the turbine parameters, aspect ratio and velocity ratio, respectively. To characterize the flow through the
turbine, Reynolds number, \(Re\), was calculated by applying the density, \(\rho\), dynamic viscosity, \(\mu\), outer velocity, \(v_o\), and hydraulic diameter = \(2b\), appropriate for flow through a wide rectangular duct (Munson, et al., 2002):

\[
Re = \frac{\rho v_o 2b}{\mu}
\]  

(8)

Equivalent roughness value, \(\varepsilon = 0.045\) mm, for commercial steel was selected. Referencing the Moody chart, a corresponding value of \(f = 0.025\) was obtained (Munson, et al., 2002).

Rice (1965) had observed that with decreasing \(Q/\Omega r_o^3\), efficiency was decreased. It is to be noted that torque, \(T\) and power, \(P\) are also functions of flow rate parameter:

\[
T = -(v_i r_i - v_o r_o) Q \rho
\]  

(9)

\[
P = \eta T \Omega
\]  

(10)

For the design, \(Q/\Omega r_o^3 = 0.0001\) was selected to maintain an appropriate level of efficiency while producing appreciable levels of power.

With decreasing \(\Omega r_o/\nu_o\), required pressure was seen to increase, in turn lowering efficiency. A design value of \(\Omega r_o/\nu_o = 1.00\) was chosen to manage required pressure.

Figure 1 is a plot of the turbine performance using the turbine parameters selected for the design. The peak efficiency is 79.7% with \(x = 0.238\), \(y = 0.394\), and \(\Delta p/\rho \Omega^2 r_o^2 = 1.137\). With all dimensionless values determined and design values for \(r_o\) and \(\Omega\) selected, remaining design values were calculated: \(r_i = 0.0595\) m, \(v_i = 5.156\) m/s, \(Q = 0.0818\) L/s, and \(\Delta p_i = 195\) kPa (equivalent to approximately 20 m head). Referring to Eq. (9) and (10), \(T = 0.242\) N·m and \(P = 10.116\) W were calculated. For 300 W capability, approximately 31 disks would be required. It is to be noted that the disk thickness is arbitrary, however a thickness that is strong enough to prevent warping while being cost and weight efficient and easily workable must be considered.

The design values have been summarized in Table 1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Measure</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer radius, (r_o)</td>
<td>250</td>
<td>mm</td>
</tr>
<tr>
<td>Inner radius, (r_i)</td>
<td>59.5</td>
<td>mm</td>
</tr>
<tr>
<td>Disk spacing, (b)</td>
<td>5</td>
<td>mm</td>
</tr>
<tr>
<td>Angular velocity, (\Omega)</td>
<td>500</td>
<td>rpm</td>
</tr>
<tr>
<td>Outer velocity, (v_o)</td>
<td>13.090</td>
<td>m/s</td>
</tr>
<tr>
<td>Inner velocity, (v_i)</td>
<td>5.156</td>
<td>m/s</td>
</tr>
<tr>
<td>Flow rate, (Q)</td>
<td>0.0818</td>
<td>L/s</td>
</tr>
<tr>
<td>Total flow rate, (Q_t)</td>
<td>2.5</td>
<td>L/s</td>
</tr>
<tr>
<td>Total required pressure, (\Delta p_i)</td>
<td>195 (20 approx.)</td>
<td>kPa (m)</td>
</tr>
<tr>
<td>Torque, (T)</td>
<td>0.242</td>
<td>N·m</td>
</tr>
<tr>
<td>Power, (P)</td>
<td>10.116</td>
<td>W</td>
</tr>
<tr>
<td>Efficiency, (\eta)</td>
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<td>-</td>
</tr>
<tr>
<td>Number of disks</td>
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<td>units</td>
</tr>
<tr>
<td>Disk thickness</td>
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<td>mm</td>
</tr>
<tr>
<td>Overall height</td>
<td>500</td>
<td>mm</td>
</tr>
<tr>
<td>Overall width</td>
<td>500</td>
<td>mm</td>
</tr>
<tr>
<td>Overall length</td>
<td>500</td>
<td>mm</td>
</tr>
<tr>
<td>Estimated mass</td>
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<td>kg</td>
</tr>
</tbody>
</table>

### VIII. Discussion

The preliminary design approach used here suggests it is feasible to design a Tesla turbine for pico hydro applications. It is to be noted that the peak efficiencies achieved in this analysis are relatively high compared to the documented overall efficiencies of past Tesla turbine investigations. As stated in the assumptions, losses due to flows in the inlet nozzle and exhaust are not considered in this analysis and likely make up for the shortfall seen. That said, this analysis can still be a useful tool towards optimizing the turbine design when used in a comparative manner.

Total required pressure for the preliminary design was in the higher region for pico hydro systems which is in the realm of impulse type turbines, such as Pelton or Turgo wheel designs. These designs are established technologies...
and popular amongst users. The implication of operating under high head limits the application to fewer sites that have water flows that can match this requirement.

Lines power has frequencies of 50 Hz or 60 Hz, depending on regions. 4-pole generators are typically employed and must be driven at an angular velocity of 1500 rpm to achieve 50 Hz. The rotational velocity, $\Omega = 500$ rpm, of the preliminary design will require either a transmission connecting the turbine to the generator to increase rotational speeds of the generator shaft, or a generator with an increased number of poles, i.e. 12-pole generator. Both options will increase complexity of the system and result in higher costs.

There is no argument that the design is relatively simple, however to achieve appreciable power generation, larger sized components are required. Material costs will increase due to this, as will manufacturing costs since equipment must be sized to handle these components.

IX. Conclusions

The nature of this design is preliminary and takes into consideration numerous assumptions. A Matlab program was devised to extend the work of Rice’s analytical investigation into Tesla turbines. Employing this, a preliminary design of a Tesla turbine has been conceived to generate 300W under conditions of 20 m head and 2.5 L/s flow. The overall dimensions of the unit will be contained by a cube 0.5 m on each side and consist of 31 co-rotating disks. The efficiency of the preliminary turbine design was near 80% but it is believed that efficiency losses will be found in inlet and exhaust flows, which were not considered in this analysis. Challenges still remain for the application of Tesla turbines to pico hydro generation and were highlighted throughout this preliminary design process. As mentioned, overall efficiencies are relatively low with respect to other small hydro systems. Focus is needed on improving inlet nozzle and exhaust designs. It was found that the Tesla turbine requires high levels of head and places the system in competition with the well established Pelton and Turgo wheel designs. Rotational velocities from the preliminary design require a transmission or an increase in generator poles to achieve lines power. Component sizes were large and will increase material and manufacturing costs. In closing, it is feasible for Tesla turbines to be used in the pico hydro context. However, much work must be done to improve the viability of Tesla turbine technology.

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References


